Exercise 1.   Efficient size reduction preprocessings for CNF.

Let p_1, p_2 be two preprocessings for CNF formulae. p_1 is at least as efficient as p_2 w.r.t. size reduction if and only if for every CNF Σ, every clause of $p_2(\Sigma)$ is subsumed by a clause from $p_1(\Sigma)$. Show that vivification (VI) is strictly more efficient w.r.t. size reduction than subsumption elimination (SE).

Exercise 2.   Encoding of graph coloring

Let $G = (V, E)$ be a graph. Find a CNF formula F_G that can be constructed in polynomial time and that is satisfiable if and only if G is k-colorable, with k given in parameter.

Exercise 3.   Diner at a "bouchon".

Dédé & Nénef plan to go out this evening for diner at a "bouchon" in Lyon. The menu at the restaurant they have chosen is as follows:

- Starter:
    - cervelas lyonnais (300)
    - pâté en croûte (300)
    - salade verte (20)

- Main course:
    - quenelle de brochet (250)
    - tablier de sapeur (360)
    - gâteau de foie de volaille (170)

- Dessert:
    - tarte aux pralines (300)
    - cervelle de canut (100)
    - salade de fruit (100)

In the menu, each item is associated with the corresponding number of calories. For instance, "cervelas lyonnais" counts for 300 calories. Dédé & Nénef are hungry so they would like to get at least 400 calories from the diner, but since they want to be in a pretty good shape tomorrow to attend the research school on knowledge compilation, they want to upper bound this number by 800.

Furthermore, they do not want to eat any of the following combinations:

- "tablier de sapeur" and "tarte aux pralines"

- "cervelas lyonnais" and "tablier de sapeur"

- "salade verte" and "salade de fruit"

1. Encode the above problem into a `CNF` formula and store it in file menu.cnf respecting the following DIMACS format:

   - The first line of the file is of the form `p cnf #var #cl` where `#var` stands for the number of variables of the instance and `#cl` stands for the number of clauses

   - Any line starting with a "`c`" is a comment line

   - The other lines represent the clauses of the instances: each positive literal x_i is encoded as a positive integer `i` and its complementary literal as `-i`, each clause corresponds to a sequence of literals separated by spaces and finishing by a `0`

   - For instance:
     ```
     p cnf 2 3
     1 -2 0
     -1 2 0
     1 2 0
     ```
     represents the `CNF` formula $(x_1 \vee \neg x_2) \wedge (\neg x_1 \vee x_2) \wedge (x_1 \vee x_2)$

2. Use d4 for determining how many plate combinations are feasible given the above constraints

3. Determine the number of plate combinations including the choice of "quenelle de brochet" for the main course

Exercise 4. Complexity of semantical decomposition.

Prove that the problem of deciding whether a given bipartition (X_1, X_2) of $Var(\Sigma)$ induces a semantical decomposition of a `CNF` Σ is coNP-complete.